Estimation of Rail Branch Line Cost Functions

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Some fundamental problems relating to the rural transportation system in both Canada and the United States stem from their extensive rail branch line networks. Part of the problem relates to their deteriorated condition. Several factors have contributed to the excess capacity in the branch line systems. These include initial over-expansion, reluctance to adapt to changing technology, and the political process of branch line abandonment.[1]

Since the early 1960's, the government of Canada has become increasingly concerned with a possible resolution of branch line problems. Alternative solutions proposed by the Canadian government include public decision-making in the abandonment/retention decision and public expenditures to rehabilitate or upgrade branch lines. The problems in the U.S. are similar, and policies have been legislated in an effort to resolve them. Both the Railroad Revitalization and Regulatory Reform Act and the recent Task Force on Rural Transportation have addressed these problems. Proposed alternatives include partial federal funding of rehabilitaion and upgrading and the relaxation of abandonment procedures.[2]

A possible solution tothe problem is to revise the rate structure to include an explicit rate for branch line service, in addition to the through rate. At present, there is little or no difference between rail rates from branch line points of origin and those at nearby mainline points of origin. Under this solution, users would evaluate the cost of branch line service (relative to the

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cost of trucking to mainline elevators) and make the appropriate modal decision. In other words, decisions regarding the economic desirability of a branch line would depend on its value as determined jointly by its users.[3]

In the public decision making process, the costs of providing branch line service are normally (implicitly, or explicitly) compared to the potential benefits of branch line retention. However, equally important to this question is the functional relationship between branch line costs and output. Estimates of cost functions are necessary in order to calculate the marginal cost and the elasticity of cost with respect to output and to examine the effect of changes in the level of output.

The general purpose of this paper is to discuss the procedures and results of the estimation of rail branch line cost functions. Using data from branch lines in Western Canada, nonlinear and polynominal cost functions are specified and estimated. The procedures are generally applicable elsewhere.

The paper commences with a brief review of branch line related studies and studies on estimating railroad cost functions. In subsequent sections, the model is specified and the empirical procedures and results are discussed. Conclusions are drawn and policy implications are discussed.

Rationalization Studies

Rationalization refers to adjustments in the size or components of a plant or system so that the same output can be produced with fewer resources. In the grain handling and transportation system, rationalization generally refers to abandonment of uneconomic branch lines, longer distances for delivery by farm truck, and fewer elevators — each with a greater throughput. The effect of rationalization in the grain handling and transportaion system has been addressed in several studies which have incorporated these interdependencies. Components of the system include trucking costs to primary or subterminal elevators, economies of density in the primary elevator, and railroad costs from the originating elevator to the terminal market. As branch lines are abandoned, truck hauling distances and costs increase, fewer elevators operate (each with greater throughput) and railroad costs decrease. Tyrchniewicz and Tosterud simulated branch line abandonment to analyze its effects in the Boissevain Region of Manitoba.[4] The system was comprised of farm to country elevator gathering costs, elevator costs, and railroad costs from that region to Thunder Bay. A similar study by Baumel, Miller and Drinka simulated the effect of branch line abandonment in Iowa and calculated benefits and costs of abandonment.[5] In both cases, average costs were assumed constant and derived from aggregated data. Further, by nature of the cost information used, the effect of branch line abandonment on rail costs was not examined. If railroad cost functions had been used, as branch

lines were abandoned (or deleted in the simulation model) and traffic diverted to nearby lines, rail unit and marginal costs on those retained would change, unless they were constant with respect to changes in output. If the cost functions for rail branch line service exhibited either increasing or decreasing economies of density, unit costs on the retained lines would change accordingly.

A more recent study by Harris simulated alternative scenarios to determine the economic viability of branch lines in the United States.[6] Waybill data were used in that study. The costs used were approximations from six previous studies, each individually suffering from aggregation problems.

Railroad Cost Function Studies

Recent studies have reported results of cost function estimations for operating both Class I and Class II U.S. railroads. The studies in general address very broad transportation issues, and at least one implicitly addresses the branch line problem.

The procedures and results of a study by Keeler are important because the relationship between short and long run cost functions was distinguished.[7] Neo-classical production functions were assumed for freight and passenger service from which a nonlinear short run cost function was derived. It was estimated directly using cross-section data from Class I U.S. railroads. The optimum number of track miles (a proxy for size) was derived for given traffic level using the parameters of the short run function. A long run cost envelope was derived by substituting the optimum number of track miles into the cost equation. The procedure was innovative, because it explicitly distinguished between economics of density and economies of scale. Both cases refer to decreasing unit costs. In the former, unit costs decrease with increases in output, but the plant size is constant. In the latter case, the plant size also increases as unit costs decrease. The results indicated that there were substantial economies of density, but the returns to scale were constant.

Economies of density were the primary concern in three subsequent studies. Harris estimated a cost function using cross-section data for Class I U.S. railroads.[8] Initially, a linear total cost function was specified but, because of heteroscedastic residuals, it was adjusted and reinterpreted as a nonlinear unit cost function. The independent variables in the adjusted function were the reciprocal of the average length of haul and density. The results indicated that there were significant economies of density in railroad freight operations. The majority of the economies were attributable to high operating costs per mile rather than high capital costs.

The cost function specified by Harris was also estimated by other authors using data from Class II U.S. railroads, in an effort to address branch line problems. Specific data on branch line operations were not available. Time series data were used in one study to estimate short run functions.[9] The results indicated that unit costs exceeded marginal costs, thereby implying economies of density. The same authors in a second study used cross-section data to estimate longer run functions.[10]

In general, cost function studies for Class I railroads have addressed broad transportation policy concerns. The procedures used by Keeler were innovative but suffered from aggregation problems. Data are usually available only at the firm level and are aggregated over many types of outputs and traffic densities.[11] Consequently, extrapolation of these results to specific firm operations would be misleading. Specific branch line data were used in this study to estimate cost function for that service, thereby reducing the aggregation problem.

Model Specification and Econometric Procedures

Recent developments in cost analysis have stressed the relationship between cost functions and their underlying production relationships.[12] Assuming the firm's objective is to minimize costs, cost functions of the following form can be derived from production functions:

$$C^* = c(Q, P_1, P_2, \ldots P_n)$$

where C^* is the minimum total cost of producing output Q, and P_j (j = 1, 2...n) is the price of n inputs. Shephard's Duality Theorem states that there exists a "well behaved" production function corresponding to every "well behaved" cost function. Production parameters may be estimated from either a production or cost function provided the latter is positive, homogenous, nondecreasing and concave in factor prices (i.e., well behaved).[13] The precise form of the cost function depends on the assumed form of the production function.

However, application of this approach to cost function specification depends on the maintained assumption of cost minimization by the firm. It is unlikely that the railways in Western Canada have an incentive, or are able, to minimize the cost of branch line operations. This contention is supported by a subsidy program which reimburses the railways for the "actual loss" incurred in branch line operation, and the rigidities and regulations over branch line retention and maintenance imposed on railway management. The production function approach to cost function specification also assumes variations in input prices across observations. The unit of observation in this study is the

branch line, each of which is operated by one of two railroads. Consequently, it is unlike that factor prices vary across observations. Further, it was not possible to discern input utilization or factor prices from the data. For these reasons, the recent developments in cost function specification were not applicable and two general cost functions were specified and estimated.

A polynomial function was specified in the following general form:

$$C_i = \beta_0 + \beta_1 M_i + \beta_2 Q_i + \beta_3 Q_i^2 + \beta_4 Q_i^3 + ei$$

where C_i is the total cost of operating branch line i, M_i is its length measured in miles and Q_i is its output measured in car-miles. A cost function which was nonlinear in parameters was also specified and estimated. The function was the following form:

$$C_i = \gamma_1 M_i + \gamma_2 Q_i \; \gamma_3 + e_i$$

where γ_1, γ_2, and γ_3 are the coefficients to be estimated. The advantage of this specification is that it stimultaneously allows for a nonlinear functional form and elasticities which vary throughout the range of output.[14]

There are several methods for estimating the parameters of the inherently nonlinear cost function. In each case the sums of squared residuals are minimized but their underlying computational procedures differ.[15] The Gauss method was used in this study which is an iterative procedure requiring starting values for the parameters to be specified. The function is expressed as a Taylor series expansion and, using the starting values for the coefficients, the sum of squared residuals are minimized to derive new coefficients values. The process is repeated using the new values until the coefficients converge according to a specified criterion.

There are two problems in the estimation of equations which are nonlinear in parameters. First, it is not certain whether the estimated coefficients so derived are from the global, or a local, minimum of the sum of squared residuals. To ensure that the minimum was global, the model was reestimated using different starting values for the coefficients.[16] Another inherent problem in nonlinear parameter estimation is that, because the residuals are not normally distributed, standard statistical methods are not appropriate to test the significance of the equation or the coefficients. However, asymptotic standard errors were calculated as a test of significance.

Regression Fallacy Bias

A potential serious bias which may occur when estimating cost functions from cross-section data is called the regression fallacy bias which is due to the stochastic nature of expenditures and output across observations.[17] More specifically, it arises because the observed output rate deviates from the planned rate due to unforeseen changes in demand. The regression fallacy results in biased coefficient estimates and is common in estimation of railroad cost functions because of the sporadic nature of capital expenditures and demand. Several methods have been suggested for reducing its effect. Borts segmented his observations by size and estimated a cost function for each sample.[18] Another method, suggested by Meyers and Kraft, is to minimize the sum of squared residuals subject to the constraint that the residuals are greater than zero.[19] The resulting function is a long run planning curve, but statistical tests for evaluating the estimates are not available. The most common method to reduce the effect of extreme years is to use cost and output data averages over several years. In this study, the cost and output data were averaged over the three year period from 1972 to 1974. Covariance analysis was used to ensure that the differences in the coefficients obtained from each of the individual years were the same, and in all cases they were. The cost functions were also estimated using pooled data for comparison but only the results of the averages data are presented.

Homoscedasticity

Another common problem in the estimation of cost functions is that error terms are frequently heteroscedastic.[20] Harris deflated each variable for each observation by output to correct for the heteroscedastic error terms.[21] This was done without estimating to which variable the heteroscedasticity was related. The reason Harris deflated by output was that he was primarily interested in the relationship between average costs and density. Consequently, the function was reinterpreted as a unit cost function as opposed to interpreting the coefficients as more efficient estimates of the total cost function.

An alternative method of adjusting the data, as used in this study, is to first estimate the form of heteroscedasticity and then deflate the date accordingly. The advantage of this method is that it does not require *a priori* assumptions about the form of the heteroscedasticity and to which variable it is related. To determine the appropriate deflator, the absolute value of the residuals obtained from the original estimated equation were regressed on various powers and combinations of the independent variables.[22] The form of heteroscedasticity, and consequently the appropriate deflator, was selected from the various estimated equations according to the coefficient of

determination and the significance of the estimated parameters. The data were transformed and the model was reestimated. The coefficients obtained from the transformed model are more efficient that those obtained from the original model. However, they maintain the same economic interpretation as in the original function.[23]

Traditional procedures for testing heteroscedasticity are inappropriate in nonlinear regression analysis because the residuals are not normally distributed. In this study, a method following Keeler utilizing the Mann-Whitney U-test was used to test for heteroscedasticity and to select the appropriate deflator.[24] The following model was estimated:

$$\frac{Ci}{M_i^W} = \gamma_1 \frac{Mi}{M_1^W} + \gamma_2 \frac{Qi\ \gamma^3}{M_i^W}$$

It was estimated for values of w scanned from 0 to 1 by .25. In each regression, the residuals from the smallest and largest N observations were compared to test if they came from the same populations. The test was applied to the residuals for each of the values of w. Heteroscedasticity was deemed minimized as soon as it was found that the smallest and largest residuals came from the same population. For the results presented here, the value of w was 0, which implies that the residuals were homoscedastic.

Short Run and Long Run Functions

Empirical distinctions are sometimes made between long and short run cost functions by the type of date used. It is traditionally assumed that cross-section data over many firms typify a long run situation whereas, time series is normally assumed to approximate the short run. The underlying logic for this empirical distinction was made by Meyer.[25] Two of the recently published studies employed this methodological concept. In one study, a model was specified and estimated using time series data and cost functions were labelled short run.[26] In another study, a function was specified where unit costs were a function of density and was estimated using cross-section data.[27] The underlying assumption when estimating cost as a function of density is that plant size is held constant. However, the estimated function was labelled long run, which implies the size of the plant can be adjusted. The confusion arises from using a restrictive form of heteroscedasticity and the concept that cross-section data typifies a long run function.

However, whether estimated cost functions as long run or short run can also be discerned by the institutional structure within which the industry operates.

A prerequisite to estimation of a long run function is the ability to choose and adjust to the optimum size of the plant. Railroads, due to their common carrier status, have not been free to adjust the size of their plant. This is especially the case with government control over branch lines. The cost functions estimated in this study are short run and the resultant economies are economies of density, rather than scale.

Data Source

The data used in this study originated from railroad submissions to hearings of the Commission of Grain Handling and Transportation (Hall Commission). During the inquiry, local hearings were held for all of the branch lines in Western Canada not protected to the year 2000. The railroads submitted detailed cost and operating data for each branch line. Components of total cost included labor, other, road maintenance, and capital costs. Other expenses included material used in the operation and maintenance of equipment as well as an account for depreciation and return on investment for those assets.

Capital costs included both depreciation and cost of money. Depreciation was zero for most branch lines because of the old age of the assets. Cost of money was the product of the cost of capital and the value of the assets. The before-tax cost of capital was 20.8 percent which was sufficient to yield an 11.31 percent after-tax return. These were determined by an independent study on railroad costing in Canada.[28] The value of the asset was the net book value. The average values were $10,576 and $15,013 per mile for CN Rail and CP Rail branch lines respectively. Generally, CP Rail Branch lines were valued greater than those for CN Rail because of their larger carrying capacity.

Transportation output is measured by the product of the volume of freight and the distance it is carried. Output is usually measured in ton-miles which incorporate both weight of traffic and distance. If data are available, a distinction can also be made between loaded and empty car-miles.[29] In this study, by nature of the data, loaded car-miles were used as the measure of output. It was measured as:

$$Q_i = \sum_{j=1}^{n} (L_j \cdot D_j)$$

where D_j is the distance from the jth shipping point to the main line junction, L_j is the number of carloads originating from that point and N is the number of shipping points on each line.

The length of branch line is measured in miles and is included in the function to account for variations in cost which are independent of the output level. The branch line length is a fixed factor of production in Western Canada because abandonment has been restricted by the political process.

Results

The data were initially analyzed to determine if the estimated branch line cost functions differed significantly between the two major railroads operating in Western Canada. Using the polynominal function, the Chow Procedure was used to test if the cost functions for the two railroads were significantly different. In all cases they were and in the subsequent analysis regressions were estimated for each of the two railroads.

The polynominal function was initially specified and estimated as a third order polynominal. The results indicated that the third order term, as well as the intercept, were statistically insignificant. These terms were deleted and only the results of the second order polynominal are reported here. The interpretation of a statistically insignificant intercept is that common costs (those which are independent of the output level and mileage) are not significantly different than zero. Its exclusion, however, is consistent with the concept of mileage related costs where that variable (M) accounts for the variation in costs which are related to provision of the capacity. The third order term was included to account for eventually increasing marginal costs but in all cases it was not significantly different from zero. The interpretation of excluding the third order term is that within the range of observations marginal costs do not increase at greater output levels. This does not deny that marginal costs may eventually increase (i.e., a capacity constraint may be reached but only at higher output levels.

The estimated coefficients for the polynominal cost function are shown in Table 1. All the coefficients have the expected sign and are statistically significant. The results indicate that total costs are positively related to output but increase at a decreasing rate. This implies that average costs are decreasing over the observed range of output. The results also indicate the mileage related costs are $4,609 and $3,355 per mile for CN Rail and CP Rail respectively. These are the costs which are independent of the output level and are associated with branch line maintenance, capital cost, taxes, etc.

The estimated coefficients for the inherently nonlinear cost functions are shown in Table 2 with their respective asymptotic errors in parentheses. All coefficients have the expected sign and, in the case of γ_1 and γ_3 are substantially larger than their asymptotic standard errors. In order to test and adjust this model for heteroscedasticity, each component of the equation was deflated by MiW, where w was scanned from 0 to 1 by .25. At each of thefive values of w, the absolute value of the residuals of the smallest N and the largest N observation were compared using a rank-sum statistic. The null hypothesis was that the absolute value of the residuals in the two samples are from the same population. Heteroscedasticity was assumed minimized for that value of w as soon as the hypothesis was accepted. In all cases, the hypothesis was accepted where w equals zero.

The results are similar to those in the polynominal function. γ_1 indicates those costs which are independent of output and γ_2 and γ_3 indicate that total costs are increasing at a decreasing rate. A comparison is made of the estimated costs and elasticities at the average value of the independent variables for the two estimating equations in Table 3. The estimated total costs are shown along with the average total cost, the marginal output costs, and marginal line related costs. The marginal output cost (dC/dQ) is the additional cost of producing a car-mile of service at the average level of output. The marginal line related cost (dC/dH) is the change in total cost associated with owning and maintaining one mile of branch line. In all cases, the elasticity of cost with respect to output is less than unity, indicating the extent marginal costs are less than average total costs.

TABLE 1

COEFFICIENT ESTIMATES FOR THE SECOND-ORDER POLYNOMINAL COST FUNCTION

Railroad	1	2	3	Adjusted R^2	N
CN Rail	4,609* (701)	3.33* (1.11)	-0.00000849** (0.00000354)	.93	42
CP Rail	3,355* (262)	3.32* (0.53)	-0.00001107* (0.00000270)	.98	22

Standard errors appear in parentheses below the regression coefficient. An * indicates significance at the one percent level; ** indicates significance at the five percent level.

$$\text{Model: } \frac{C}{x} = \beta_1 \frac{M}{x} + \beta_2 \frac{Q}{x} + \beta_3 \frac{Q^2}{x}$$

where the β's are the estimated coefficients and x is the deflator.

TABLE 2

COEFFICIENT ESTIMATES FOR THE NONLINEAR COST FUNCTION

Railroad	1	2	3	N
CN Rail	4,206 (1,095)	667.30 (1,019.86)	0.51 (0.11)	42
CP Rail	4,176 (559)	10.60 (33.56)	0.82 (0.25)	22

Asymptotic standard errors appear below the regression coefficient.

$$\text{Model: } C_i = \frac{\gamma_1 M_i}{M_i^W} + \frac{\gamma_2 S^3}{M_i^W} \qquad \text{where } w = 0$$

γ_i's are the estimated coefficients.

A comparison of the relative efficiency of the two railroads from this table is inappropriate because the costs are estimated at the average value of the explanatory variables for each. The average branch line length and output is 55.1 miles and 53,666 car-miles for CN Rail as opposed to 44.0 miles and 36,623 for CP Rail. The estimates of costs and elasticities were calculated at the same level of branch line output and length and are shown in Table 4. The branch line length was assumed to be 50 miles with an output of 48,000 car-miles. These values were used because they are the approximate mean of the averages of the variables for the individual railroads. In both cases the estimated total costs, marginal output costs and marginal line-related costs for CN Rail were greater than those calculated for CP Rail. A general explanation for these cost differentials is related to the age and structural condition of the branch lines. Generally, CP Rail branch lines are relatively newer, have heavier steel and are in better structural condition than those of CN Rail.

Conclusions

A fundamental problem in the rural agricultrual rail system in both Canada and the United States is their extensive and deteriorating branch line systems.

Political constraints have aggravated the problem by not encouraging the trend towards the abandonment of uneconomic branch lines, despite economic pressures for a rationalized system. The functional relationship between the cost of operating branch lines and output is fundamental information necessary in the decision-making process and in system simulations.

Cost functions for rail branch line service in Western Canada were specified and estimated in this study. Data were assembled across branch lines for both the major railroads. Total cost included all the capital and variable costs associated with providing branch line service with the exception of rehabilitation costs.[30] Output was measured as car-miles and length in miles. The general model was estimated using both a polynominal and nonlinear in parameters cost function. The estimated coefficients were inefficient in several cases due to heteroscedastic residuals but were adjusted accordingly. The economic and statistical criteria were acceptable in both models.

TABLE 3

ESTIMATED COSTS AND ELASTICITIES USING THE AVERAGE VALUES OF THE INDEPENDENT VARIABLES

Railroad	Total cost ($)	Average total cost ($/car-mile)	Marginal output cost ($/car-mile)	Marginal mileage cost ($/mile)	Elasticity of cost with respect to output
CN Rail					
Polynomial	408,471	7.61	2.42	4,609	.32
Nonlinear in parameters	404,123	7.53	1.64	4,206	.22
CP Rail					
Polynomial	254,455	6.95	2.59	3,355	.36
Nonlinear in parameters	242,302	6.62	1.31	4,176	.20

1. The average values were 55.1 and 44.0 miles and 53,666 and 36,623 car-miles for the Canadian National and the Canadian Pacific respectively.

2. Capital funds rate equals 20.8 percent.

TABLE 4

COMPARISON OF ESTIMATED COSTS ASSUMING A 50 MILE BRANCH LINE WITH OUTPUT OF 48,000 CAR-MILES

Railroad	Total cost ($)	Average total cost ($/car-mile)	Marginal output cost ($/car-mile)	Marginal mileage cost ($/mile)	Elasticity of cost with respect to output
CN Rail					
Polynomial	370,936	7.73	2.52	4,609	.33
Nonlinear in parameters	373,137	7.77	1.73	4,206	.22
CP Rail					
Polynomial	301,766	6.29	2.26	3,355	.36
Nonlinear in parameters	281,911	5.87	1.25	4,176	.21

1. Capital funds rate equals 20.8 percent.

The results indicate that a large proportion of the cost of operating branch lines is independent of the output level. As a result, both average and marginal costs decrease over the observed range of output. These conclusions have important implications for the formulation of public policy affecting branch lines. As branch lines are abandoned and traffic is diverted to nearby branch lines, unit and marginal costs on the retained line would decrease. This would be in addition to the opportunity costs of retaining the line which was abandoned. Normally, only the latter effect is incorporated into models of grain handling and transportation. Inclusion of the former effect would also be useful in simulation and policy analysis regarding branch line abandonment.

The conclusions are also important in the event that specific rates for branch line service, in addition to through rates (i.e., a branch line surcharge), are proposed. The general purpose of such a proposal would be to allocate specific costs to specific users. Rates equal to the unit cost of branch line service would result in revenue equal to the total cost of producing the service. However, such a rate structure would be ineffecient because some users willing to pay the marginal cost, but not average cost, would be excluded.

Prices are efficient if they equal marginal cost. The results indicate that, if branch line rates were uniformly equal to marginal cost, approximately 20 to 35 percent of the total cost of producing the service would be recouped. A subsidy would be necessary to finance the deficit and would be justified if the marginal benefits exceeded the marginal cost of producing additional service. An alternative to an external subsidy is for prices to be differentiated across users. Optimal departures from the marginal cost pricing rule result in prices inversely related to the elasticity of demand for the service.[31]. This latter alternative poses administrative difficulties but it does allocate resources effectively over the long run.

FOOTNOTES

1. For further description of the problem in Canada see Report of the Grain Handling Transportation, *Grain and Rail in Western Canada,* Vol. I (Ottawa: Government of Canada, 1977), pp. 33-43.

2. United States, Department of Agriculture and Department of Transportation, *Agricultural Transportation Services Need, Problems, Opportunities,* The Final Report of the Rural Transportation Advisory Task Force (January 1980), pp. 21-23.

3. The concept of pricing of branch line services alluded to is fully developed in W.W. Wilson and E.W. Tyrchniewicz, "A Conceptual Framework of Financing Rail Branch Line Operation and Rehabilitation," in the Proceedings of the World Conference on Transport Research, *Transportation Research for Social and Economic Progress,* London, April 1980.

4. E. W. Tyrchniewicz and R.J. Tosterud, "A Model for Rationalizing the Canadian Grain Transportation and Handling System on a Regional Basis," *American Journal of Agricultural Economics,* Vol. 55, No. 5 (December 1973), pp. 805-813.

5. C.P. Baumel, J. Miller and T. Drinka, "The Economics of Upgrading Seventy-One Branch Rail Lines in Iowa," *American Journal of Agricultural Economics,* Vol. 59, No. 1 (February 1977), pp. 61-70.

6. R.G. Harris, "Economic Analysis of Light Density Rail Line," *The Logistics and Transportation Review,* Vol. 16, No. 1 (1980), pp. 3-31.

7. T.E. Keeler, "Railroad Costs, Returns to Scale and Excess Capacity," *Review of Economics and Statistics,* Vol. LVI, No. 2 (May 1974), pp. 201-208.

8. R.G. Harris, "Economics of Traffic Density in the Rail Freight Industry," *The Bell Journal of Economics*, Vol. 8, No. 2 (Autumn 1977), pp. 556-564.

9. A. Charney, N. Sidhu and J. Due, "Short Run Cost Functions for Class II Railroads," *The Logistics and Transportation Review,* Vol. 13, No. 4, (1977), pp. 345-359.

10. N. Sidhu, A. Charney and J. Due, "Cost Function of Class II Railroads and the Viability of Light Traffic Density Rail Way Line," *Quarterly Review of Economics and Business,* Vol. 17, No. 3 (1977), pp. 7-24.

11. Keeler, *op. cit.,* pp. 208.

12. H.R. Varian, *Microeconomic Analysis* (New York: W.W. Norton, 1978), pp. 123-127.

13. Problems with this approach have been discussed. In particular, Burgess noted that application of the translog cost (or production) function is very sensitive to the data. This functional form is actually a form of a truncated Taylor's series (i.e., a second order approximation) which may not perform uniformly throughout the entire data range. When a translog cost and translog profit function were applied to the same phenomena, they failed to yield uniform parameter estimates. The theory suggests, and duality requires that, the optimal output and input combinations are the same for cost minimizing as for profit maximizing. See D. Burgess, "Duality, Theory and Pitfalls in the Specification of Technologies," *Journal of Econometrics,* Vol. 3, No. 2 (1975), pp. 105-121.

14. As opposed to a log linear cost function which is restricted to a constant elasticity.

15. R.S. Pindyck and D.L. Rubinfeld, *Econometric Models and Economic Forecast,* (New York: McGraw-Hill, 1976), pp. 225-234.

16. This method was suggested by Pindyck and Rubinfeld, *Ibid, p. 229.*

17. For a thorough discussion of the regression fallacy, see G. H. Borts, "The Estimation of Rail Cost Functions," *Econometrica,* Vol. 28, No. 1 (January 1960), pp. 108-131.
20.

18. *Ibid.*

19. J.R. Meyer and G. Kraft, "The Evaluation of Statistical Cost and Techniques as Applied in the Transportation Industry," *American Economic Review: Papers in Proceedings,* Vol. 51, No. 2 (May 1961), p. 325.

20. Tests were also used to examine the effects of collinearity between independent variables. In all cases the coefficients were stable when variables or observations were deleted.

21. Harris, "Economics of Traffic Density in Rail Freight Industry," *op. cit.*

22. H. Glejser, "A New Test for Heteroscedasticity," *Journal of the American Statistical Association,* Vol. 64 (1969), pp. 316-323.

23. Several recent studies utilized the former method of adjusting for heteroscedasticity. Their results are not necessarily incorrect but their methodologies and interpretations are confusing. Harris specified a cost function and deflated it by output, rather than any of the other explanatory variables, because his "primary interest was in the relationship between average cost and density." The estimated coefficients from the transformed model were interpreted as a unit cost function which is universely related to density, rather than more efficient estimates of the original coefficients. The confusion in the interpretation of the functions originates from the heteroscedasticity being assumed *a priori* to be related to output as opposed to other explanatory variables. Further, the form of heteroscedasticity was restrictive and arbitrarily chosen. See Harris, *op. cit.* Subsequent studies estimated models identical to Harris' transformed equation and in one study the model was again adjusted for heteroscedasticity. See Charney, Sidhu and Due, *op. cit.* and Sidhu, Charney and Due, *op. cit.*

24. Keeler, *op. cit.*

25. J.R. Meyer, "Some Methodological Aspects of Statistical Costing as Illustrated by the Determination of Rail Passenger Costs," *American Economic Review: Papers and Proceedings,* Vol. 28, No. 2 (May 1958), pp. 212.

26. Charney, Sidhu and Due, *op. cit.*

27. Sidhu, Charney and Due, *op. cit.*

28. Report to the Commission on the Cost of Transporting Grain by Rail, Vol. I (Ottawa: Government of Canada, October, 1976).

29. Harris, "Economics of Traffic Density in Rail Freight Industry," *op. cit.*

30. Rehabilitation and upgrading cost estimates were also included on a per mile basis in the original study. See W.W. Wilson, "Financing the Operation and Rehabilitation of Rail Branch Lines," Unpublished Ph.D. Thesis, Department of Agricultural Economics, University of Manitoba, Winnipeg, 1980, pp. 72-74, and 121-126.

31. W.J. Baumol and D.F. Bradford, "Optimal Departures from Marginal Cost Pricing," *American Economic Review,* Vol LX, No. 3, (June 1970), pp. 265-283.